



**Department of AERONAUTICS and ASTRONAUTICS  
STANFORD UNIVERSITY**

**Eighth Semiannual Status Report**

**June 1965**

on

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**BASIC STUDIES IN SPACE VEHICLE ATTITUDE CONTROL**

in the

**Department of Aeronautics and Astronautics  
Stanford University**

under

**Research Grant NSG-133-61**

from the

**National Aeronautics and Space Administration**

This report summarizes progress during the past six months under a continuing research grant for the period September 1964 to September 1965. The initial grant is based on Ref. 1, and its continuation on Ref. 2. The research is supervised by Prof. I. Flüge-Lotz and Prof. R. H. Cannon, Jr., Principal Investigators.

A separate financial accounting will be forwarded by the University.

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## SUMMARY

During this report period the study of magnetic attitude control of spinning vehicles has been completed by Mr. Wheeler, and the work on stability analysis of relay-control systems via the direct method of Lyapunov was completed by Mr. Weissenberger. Both Mr. Weissenberger and Mr. Wheeler will receive their Ph.D. degrees on June 13, 1965. Seven studies are continuing, four on different aspects of achieving near-optimal attitude control of planet-pointing satellites, one on control of large flexible spinning vehicles, one on a new technique for trajectory optimization, and one on contributions to human pilot control. No new studies have been started in this report period.

A. STUDIES OF CONTROLLED VEHICLE BEHAVIOR  
(Studies Supervised by Professor Cannon)

1. Pilot Control of a Space Vehicle

The work on the control action feedback and vector reticle display, described in Ref. 3, has been completed.

Recent work has been concerned with determining how the performance of a human operator engaged in a closed-loop tracking task depends on which of the operator's outputs is used for feedback. The operator outputs investigated were hand position, force, myoelectric activity from a pair of muscles, and angular rotation of the eye.

During the first phase of the study, performed in the Neurology Branch of the Palo Alto Veterans Administration Hospital, operators tracked a step-function target and were subjected to step-function force disturbances. Measurements of position, velocity, and acceleration of the hand, myoelectric activity of the muscles and angular position of the eye describe the organization of a movement and relate it to activity in the central nervous system. Table I shows typical latencies (measured from time of target displacement) for each of the operator's outputs. Portions of these results are described in the paper [Ref. 4] "Silent Period Produced by Unloading of Muscle During Voluntary Contraction," by R. Angel, W. Eppler, and A. Iannone which has been accepted for publication in the Journal of Physiology. Copies have been forwarded to NASA.

Table I. Typical Latencies of Operator Outputs

Operator Output	Hand Position	Force	Myoelectric Activity	Eye Position
Typical Latency	0.35 sec *	0.23 sec	0.15 sec	0.23 sec

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\* Time to reach one-half the required displacement.

On the basis of the information displayed in Table I, it was concluded that an operator could track higher frequency signals using a pressure control stick than with the conventional displacement control stick; feedback of myoelectric signals obtained from surface electrodes positioned over the muscles, should make it possible to track still higher frequency inputs. To test this conjecture, the closed-loop frequency response was measured for four different operators using each of these three types of feedback. The frequency responses were obtained from cross-power spectral density measurements made while the operator tracked a random signal. Although the data has not been completely analyzed, it appears that feedback of myoelectric signals permits the greatest bandwidth and use of the displacement control stick results in the least bandwidth; i.e., the results obtained for random input signals agree with those obtained using step-function inputs. Future effort will be devoted to quantifying these results and determining how the remnant noise (that part of the output which is not linearly correlated with the input signal) depends on the type of feedback which is used.

Both the work on control-action feedback and display and on human-operator outputs will be incorporated in Mr. Eppler's Ph.D. thesis.

## 2. Magnetic Attitude Control of Spinning Vehicles

The magnetic attitude-control law discussed in the previous report has been examined exhaustively from the viewpoint of its application to practical vehicles; the results indicate the applicability of this control system to a variety of spinning spacecraft. This work, as well as the previously summarized results, is reported in Mr. Wheeler's doctoral dissertation, [Ref. 5], copies of which are being forwarded to NASA. This dissertation deals specifically with the attitude control of an axially symmetric, rigid, spinning space vehicle in a circular earth orbit by means of passing current through a single coil so aligned as to produce a magnetic moment along the spin axis of the vehicle.

The following are the principal contributions of the dissertation:

1. A new feedback-control law is developed for magnetic attitude control of spinning vehicles. This control law includes provisions for both position control and active magnetic wobble damping.

2. The feasibility of magnetic attitude control of spinning vehicles is demonstrated by showing (with appropriate Lyapunov functions) that the above control law produces asymptotic stability in-the-large for a satellite in a circular earth orbit of any inclination with the desired spin-axis direction inertially fixed, but otherwise arbitrary.

3. The performance of the undisturbed system with this control law is evaluated by deriving KBM-type estimates for the solutions to the equations of motion (which are time varying and, for large errors, non-linear). These results indicate the influence of the altitude, the orbital inclination and the desired spin-axis direction upon the response of the undisturbed system.

4. The large-error response of the feedback-control law developed in this study is shown, for signum control of the coil current, to compare very favorably with the response using minimal-time control programs derived by applying Pontryagin's Maximum Principle.

5. The feedback-control law is shown to be practical for a wide range of applications by general discussions of the relationship between the mechanization problem (e.g., measurement of the variables required in the control law) and the mission of the satellite.

A technical paper describing some of this work has been prepared for submission to the XVI IAF Conference. Mr. Wheeler's dissertation has been accepted by the University, and he will be receiving his Ph.D. degree this month.

### 3. Attitude Control of a Flexible, Spinning, Toroidal Manned Space Station

This study by W. B. Gevarter has considered a continuous attitude control system for controlling the direction of the spin axis of a limber, spinning, toroidal manned space station.

It was found that this space station can be thought of as an example of a more general class of problems which might be referred to as "control of flexible vehicles." If one designs a control system for a rigid vehicle the relative locations of the sensors and the control forces or moments are unimportant. However, when a flexible vehicle is considered, the relative locations of the sensors and the control forces or moments are of the utmost importance as they determine the coupling between flexible modes and the rigid mode of motion we are trying to control, and are thus crucial to system stability.

In this study simple relations have been developed as a function of these force and sensor locations, from which the system's stability, its roots, and its real-time response may be rapidly estimated for both single-axis and coupled two-axis flexible vehicle control systems.

Applying these relations to the spinning space station, locations were determined for the control forces (or moments) and sensors such that the system was stable and the flexural modes were minimally excited.

This study has been essentially completed and should soon be available as a report.

#### 4. Control of Unstable Mechanical Systems

Mention is made here of some research which has been in progress at Stanford for the past three years under NSF sponsorship, and which may be of interest because of its pertinence to vehicle control. This is the work on control of unstable mechanical systems, in which the objective has been to study the controllability of plants which are inherently unstable, where control effort is limited; and to develop effective methods for synthesizing control logic which will provide stability throughout the entire region of controllability in plants which have many outputs, but only one input available to the controller.

A number of mechanical models have been constructed and successfully controlled. The latest is a very flexible beam hinged as an inverted pendulum above a cart. The control input is torque to an electric motor which drives the wheels of the cart. Pendulum angle and strain gauge measurements of beam bending were sensed and the rigid-body motions plus two bending modes were controlled.

This most recent study is reported in the Ph.D. dissertation of Capt. J. F. Schaefer, [Ref. 6], and in a paper which has been submitted to the International Federation of Automatic Control, [Ref. 7].

B. NONLINEAR STUDIES, OPTIMAL CONTROL  
(Studies Supervised by Professor Flügge-Lotz)

1. Optimum and Suboptimal Control of the Pitch Motion of an Earth Satellite in Elliptic Orbit; Preliminary Consideration of the Linearized Roll-Yaw Motion\*

The portion of this research concerned with the relatively large-error control of the linearized pitch motion has been completed. A constant, easily realized pitch control system using gas jets has been devised based on reverse-time optimal solutions. This suboptimal control scheme was found to use on the average only about nine percent more fuel than the optimal scheme. Besides being considerably simpler than the optimal scheme, it has the additional feature that the error magnitude during control is kept smaller. Also, it was found to work equally well, with only small control-parameter changes, for orbits with eccentricities ranging from 0.05 to 0.15 and for a fair variety of satellite shapes. This system has been compared to other simple suboptimal schemes and found to be superior.

The steady-state control of the pitch motion will be accomplished using a reaction wheel since the steady-state disturbances are periodic. Work has begun to optimize this controller with respect to a criterion that takes into account both control effect and error magnitude.

The linearized yaw-roll system of equations is fourth order, coupled, and time varying. To determine the optimal control or a suboptimal control for this system by the method used for the pitch equation would be highly impractical. Thus, alternate methods of attacking this problem are being considered.

2. The Complete Attitude Control Problem for an Earth Satellite in Elliptic Orbit\*\*

In the preceding (seventh) status report the difficulties of the minimum fuel control of large attitude errors of a satellite in an

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\* Study for a Ph.D. thesis of Mr. R. Busch.

\*\* Study for a Ph.D. thesis of Mr. K. Hales.

elliptic orbit have been described. At that time it was decided to assume that future computer development will make it possible to design a control system which uses digital elements on board the satellite. After testing the suggested near-optimal control on a simple example, the main task was to write and discuss the program for the entire sixth-order system. This task is composed of three parts:

1. The complete equations of motion for the satellite in general, nonlinear form have been derived.

2. Equations of the adjoints are needed in every optimization problem. To accomplish this task a matrix of partial derivatives of the original equations of motion with respect to the state variables must be formulated. At present this matrix is being derived and programmed.

3. A general steepest descent digital program, which will accept the mathematical model of a satellite controlled in three axes by gas jets whose thrust levels are either on full or off, has been written and checked out.

[The last part can principally be used for other tasks involving discontinuous control.]

Since the optimization starts with assumption of a nominal path and improves it gradually using first-order variation, each step of the three was checked with specific examples to be sure that the rather large program will work when finally put together.

The goal is to obtain a suboptimal control law whereby the initial large attitude errors may be reduced to a low level within one or two orbits.

### 3. The Validity of Linearization in Attitude Control<sup>\*</sup>

The time optimal control problem for the attitude control of a vehicle, represented by the differential equation:

$$\ddot{x} + \sin x = u \quad |u| \leq 1$$

has been almost completely studied.

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<sup>\*</sup> Study for a Ph.D. thesis of Mr. J. L. Almuzara.

Application of Pontryagin's Maximum Principle leads to the following "bang-bang" control:

$$u = \operatorname{sgn} p_2$$

where  $p_2$  is one of the adjoint variables, given by the equation:

$$\ddot{p}_2 + p_2 \cos x = 0.$$

It has been shown, with the help of the so-called canonical paths and the fact that the zero trajectories divide the phase plane in two parts, that two is the maximum number of switchings in order to bring the system to rest after a general initial disturbance.

Mathematical expressions, rather complicated, have been found which enable us, with the help of any known numerical technique, to find both the optimal switching loci and the separation curve (points from which we can attain the origin in the same time with either 1 or 2 switchings).

A comparison was made between the above problem and the linearized one, represented by:

$$\ddot{x} + x = u \quad |u| \leq 1$$

and many discrepancies were found. The most important are:

1. Number of switchings
2. Optimal switching loci
3. Separation curve, nonexistent in the linear case
4. Different minimum times for zeroing any initial disturbance.

Attempts are being made to generalize the above results to the problem, represented by the differential equation:

$$\ddot{x} + f(x) = u \quad -B \leq u \leq A$$

where  $f(x)$  is any continuously differentiable function such that  $|f(x)| \leq \min(A, B)$ . Different kinds of nonlinearities  $f(x)$  are being considered. The problem has not yet been finished, but it promises some interesting results.

#### 4. Random Disturbances in Control Problems<sup>\*</sup>

Pontryagin's Maximum Principle offers a feasible method for the solution of control optimization problems when the disturbances affecting the performance of the system are known functions of time. At this time there is not a satisfactory "Maximum Principle" available for those cases where these disturbances are random functions of time.

When the control signal is restricted to be the output of a relay, it is shown that in some cases the stochastic control problem can be reformulated so that Pontryagin's Maximum Principle can be effectively used.

This method of solution is applied to the problem of finding a physical realization of a relay control system such that this control drives the plant to a predetermined state at a given time in the future. During the interval of control, the plant is assumed to be disturbed by gaussian noise of known spectral density. The performance criterion in this case is the minimization of the final state variance.

For those cases where the noise power is sufficiently large, a quasi-optimal feedback control scheme is offered. It is shown that this scheme approaches the true optimal scheme as the relative noise power increases.

A report about this work is nearly finished.

#### 5. Publications

The paper "On the Minimum Effort Regulation of Stationary Linear Systems" by I. Flügge-Lotz and H. D. Marbach [Ref. 8] appeared in No. 4, Vol. 279 of the Journal of the Franklin Institute. As soon as reprints are available, twenty-five copies will be sent to NASA.

Mr. Weissenberger has completed a final report, [Ref. 9] on his research, "Stability Analysis of Relay-Control Systems Via the Direct Method of Lyapunov," for which partial support was furnished from this grant.

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<sup>\*</sup> Study for the Ph.D. thesis of Mr. G. Hyver (partial support by Lockheed).

6. Computation of Approximately Optimal Control

This work, by T. E. Bullock, is concerned primarily with new techniques for the computation of optimal and suboptimal controls. The studies described in the last semiannual status report have been continued, with a strengthened emphasis on schemes based on second variations.

The method of second variations offers several advantages over conventional steepest descent procedures in trajectory optimization problems. When the process converges, the convergence is much faster than with the old method. The method also produces the neighboring optimal control scheme feedback coefficients as a byproduct without additional calculations. Also, the results give information which can be used to show that the necessary conditions are also sufficient for a local extremum.

Current experimental work is being done with computer programs for both the ordinary steepest descent and the new method in an effort to get a feel for the practical problems involved. Although the method is quite straightforward for free endpoint problems, further work is needed to clear up some of the details in the application of this idea to a problem with terminal constraints. This is the present area of theoretical interest. The next step will involve a complete computer testing of all methods on many types of problems.

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\* Professor Franklin is on sabbatical leave this year. In his absence Professor Breakwell is supervising the Ph.D. research of T. E. Bullock.

### C. OTHER ACTIVITIES

Professor Cannon was a guest lecturer at the University of Florida Third Winter Institute on Advanced Control. He participated in the AIAA Space Sciences Meeting in New York as Chairman of the Technical Committee on Guidance and Control. He also participated in meetings of the NASA Research Advisory Committee on Control, Guidance, and Navigation at Langley Research Center in December and at the Ames Research Center in May.

Professor Flügge-Lotz attended the First International Conference on Programming and Control at the United States Air Force Academy, Colorado, April 15-16, 1965.

Professor Flügge-Lotz became a member of the Nonlinear Systems Committee of the IEEE/PTGAC.

Professor Breakwell became a member of the Astrodynamics Committee of ASME. He gave an invited paper in Astrodynamics at the ASME Conference in New York in December 1964. He attended the AAS Symposium on Unmanned Exploration of the Solar System in Denver in February 1965. He also attended a Review of Research Contracts at NASA Goddard Space Flight Center, Greenbelt, Maryland in March 1965.

## REFERENCES

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2. "Request for Continuation of Research Grant for Basic Studies on Space Vehicle Attitude Control Systems," submitted to NASA by Stanford University, AERO 568, Feb 1963.
3. Seventh Semiannual Status Report, submitted to NASA by Stanford University, Dec 1964.
4. R. Angel, W. Eppler, and A. Iannone, "Silent Period Produced by Unloading of Muscle During Voluntary Contraction," to be published in the J. of Physiology.

[References 5, 6, and 9 are essentially the Ph.D. theses of the student author.]

5. P. C. Wheeler, "Magnetic Attitude Control of Rigid, Axially Symmetric, Spinning Satellites in Circular Earth Orbits," SUDAER No. 224, Department of Aeronautics and Astronautics, Stanford University, Stanford, Calif., Apr 1965.
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